ON CLOSED SURFACES IMMERSED IN $\mathbb{E}^3$ WITH CONSTANT MEAN CURVATURE

SAMUEL I. GOLDBERG AND GABOR TOTH

ABSTRACT

H. Hopf proved that a topological sphere immersed in $\mathbb{E}^3$ with constant mean curvature is a round sphere. A. D. Aleksandrov subsequently showed that the condition on the genus could be removed if the immersion were an imbedding, and he conjectured that even this condition is not necessary. However, in 1984 Wente gave an example of an immersed torus in $\mathbb{E}^3$ with constant mean curvature. More recently, Kapouleas constructed closed surfaces in $\mathbb{E}^3$ with constant mean curvature for any genus $g > 2$. In this paper, the local behavior of the Gaussian curvature K near its zero set Z is studied. Since K may be viewed as the ratio of surface elements with respect to the Gauss map $\phi: M \to S^2$, it follows that Z is the singular set of ϕ. The classification of singularities of harmonic maps given by J. C. Wood is utilized, as is an analysis of the sinh–Gordon equation to study the critical points of K on and near Z. As a consequence, the integral $\int_{\{K^2 \leq 1/8\}} \Delta K^2$, whose integrand was studied by S.-S. Chern and the first author, is shown to have interesting properties.

1. *Hopf's problem*

In 1950, Heinz Hopf proved that a closed (that is, compact without boundary) orientable surface M of genus zero immersed in Euclidean 3-space $\mathbb{E}^3$ with constant mean curvature is a round sphere S^2 [**7**, **8**]. He asked if the condition on the genus could be removed. In 1955, A. D. Aleksandrov showed that for embedded surfaces it could [**2**, **8**], and he conjectured that this was valid for immersions as well. It was not until 1984, when H. Wente [**14**] gave a striking example of an immersed torus in $\mathbb{E}^3$ with constant mean curvature, that this problem was resolved. The construction required a detailed analysis of the sinh–Gordon equation, that is, the Gauss equation (cf. also [**1**]). Using more sophisticated partial differential equation techniques, N. Kapouleas [**9**] recently gave infinitely many examples of closed surfaces in $\mathbb{E}^3$ with constant mean curvature for each $g > 2$. His construction involved piecing together slices of the classical Delaunay surfaces; this, however, does not work for $g = 2$. Note that the Gauss maps of such immersions are harmonic [**12**]. However, they are not holomorphic, and by a result of J. Eells and J. C. Wood [**6**], they have degree at most $g-1$. In this paper, the local behavior of the Gaussian curvature K near its zero set Z is analysed.

The Gauss map gives rise to a non-negative scalar invariant $C = \frac{1}{4}|\nabla\beta|^2$, where β is the second fundamental form of the immersion and $\nabla\beta$ is its covariant derivative (see §2). If $M \neq S^2$, then C and ∇K do not vanish at the smooth points of Z. (A point in Z is *smooth* if a neighborhood of it intersects Z in an arc.) This is the content of Theorem 1. It is a consequence of the sinh–Gordon equation, the superharmonicity

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of $\log K^2$ away from Z (see [**4**, p. 143]), the maximum principle for subharmonic functions, and Lemma 1, which may be looked upon as a maximum principle for superharmonic functions on a region of the Riemann sphere whose boundary consists of Jordan arcs. Theorem 1 implies that the critical points of K on Z are isolated. In addition, those critical points at which K does not have local extrema are isolated (Theorem 2).

Since the Gauss curvature is the ratio of surface elements with respect to the Gauss map $\phi: M \to S^2$, it follows that Z is the set of singular points of ϕ. The classification of singularities of harmonic maps given by J. C. Wood [**15**] and the sinh–Gordon equation imply that any non-smooth point in Z is a C^1-meeting point of an even number of general folds (Theorem 3). In particular, in Wente's example, Z is a union of figure eights, and the non-smooth points are meeting points of two general folds. Moreover, C vanishes only at the non-smooth points. In the examples of N. Kapouleas, Z consists of smooth points only and these are located on each 'Delaunay neck' which contributes two closed curves to Z. In both these examples, the condition that $K^2 \geqq 2C$ in [**4**, Proposition 3.4] is not satisfied on Z.

Finally, if $M \neq S^2$, the integral $\int_{\{K^2 \leqq 1/8\}} \Delta K^2$ whose integrand was studied in [**4**] is seen to have interesting properties. This is the content of Theorem 4. It is a consequence of a method developed by S.-S. Chern and S. I. Goldberg in [**4**], an analysis of the sinh–Gordon equation, and a classical method of Nevanlinna theory which uses an appropriate exhaustion function on $M \backslash Z$. The superharmonicity of $\log K^2$ on $M \backslash Z$ plays an essential role.

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2. *The singular set of the Gauss map and the critical points of K*

Let M be a closed surface immersed in $\mathbb{E}^3$ with constant mean curvature which is normalized to be $\frac{1}{2}$. Since this latter condition is expressed by an absolutely elliptic equation [**8**], it follows from Bernstein's theorem that M is analytic in $\mathbb{E}^3$. Hence, all data derived from the metric, such as the Gaussian curvature K, are (real) analytic. In particular, $Z = \mathit{Zero}(K)$ is an analytic set in M, and so applying the Weierstrass preparation theorem [**10**], it consists of finitely many analytic curves.

Let $\beta = (h_{ij})$, $i,j = 1,2$, denote the second fundamental form of the immersion, and let $\nabla\beta$ be its covariant derivative. Here $\nabla\beta$ is a symmetric tensor whose components are denoted by h_{ijk}. Let (H_{ij}) denote the adjoint matrix of (h_{ij}). Then $C = \sum H_{ij} H_{kl} h_{ilm} h_{jkm}$ is an (analytic) scalar invariant. Since $\phi: M \to S^2$ is a harmonic mapping by the Ruh–Vilms theorem [**12**], $\sum h_{ijj} = 0$. Thus, h_{111} and h_{112} are essentially the only components of $\nabla\beta$, so

$$C = (h_{111}^2 + h_{112}^2)(H_{11} + H_{22})^2 \geqq 0.$$

Diagonalizing β at a point, we get

$$H_{11} + H_{22} = \frac{K}{\lambda_1} + \frac{K}{\lambda_2} = \lambda_1 + \lambda_2 = 1$$

at that point, where λ_1 and λ_2 are the principal curvatures of M. Consequently,

$$C = \tfrac{1}{4} |\nabla\beta|^2.$$

Note that $C = 0$ if and only if M is a round sphere.

In the sequel, it is assumed that $M \neq S^2$. The following global formulas will be required (see [**4**, p. 143]):

$$\tfrac{1}{2}\Delta K^2 = (1-4K)(2C-K^2)-C, \tag{1}$$

and on $M\backslash Z$,

$$\tfrac{1}{2}\Delta \log K^2 = -(1-4K)-CK^{-2}. \tag{2}$$

Formula (2) says that $\log K^2$ *is superharmonic away from* Z, since $1-4K \geqq 0$ (with equality at the umbilics); this is an important fact in the proof of Theorem 4. From (1) and (2) and the identities

(i) $\tfrac{1}{2}\Delta u^2 = u\Delta u + |du|^2$, and

(ii) $\Delta \log u = -u^{-2}|du|^2 + u^{-1}\Delta u, u \neq 0$,

we obtain

$$|dK|^2 = (1-4K)C \tag{3}$$

and, on $M\backslash Z$,

$$\Delta K = -4C-(1-4K)K. \tag{4}$$

For, by (1) and (i) with $u = K$,

(iii) $K\Delta K + |dK|^2 = (1-4K)(2C-K^2)-C$,

and by (2) and (ii) with $u = K$,

(iv) $K\Delta K - |dK|^2 = -(1-4K)K^2 - C$.

Subtracting (iv) from (iii) gives (3). Substituting for $|dK|^2$ from (3) in (iii) yields (4).

It follows from (3) that $Zero(C) \subset Zero(dK)$ and $Zero(dK)\backslash Zero(C)$ consists of umbilics. In particular, $\#\{Zero(dK)\backslash Zero(C)\} \leqq 4g-4$. For, each umbilic has strictly negative half-integral index and the sum of the indices is $2-2g$, the Euler characteristic of M [**7**]. Away from umbilics, M is given locally by a conformal representation F which is determined by a solution of the sinh–Gordon equation

$$\Delta\omega + \tfrac{1}{2}\sinh 2\omega = 0,$$

where $e^{-2\omega}$ corresponds, via F, to the (positive) difference $\lambda_2-\lambda_1$ of the principal curvatures [**1**]. Since $\lambda_1+\lambda_2 = 1$, *the Gaussian curvature* K *corresponds to* $\tfrac{1}{4}(1-e^{-4\omega})$. Using this, (4) is easily seen to be equivalent to the sinh–Gordon equation.

A point $z_0 \in Z$ is said to be *smooth* if there is a neighborhood U of z_0 such that $U \cap Z$ is a single analytic arc. A non-smooth point $z_0 \in Z$ is said to be a C^1-*meeting point of* q *general folds* if a neighborhood U of z_0 is C^1 diffeomorphic to a neighborhood of the origin in $\mathbb{E}^2$ with $U \cap Z$ corresponding to $q\,(>1)$ line segments meeting at the origin [**15**; **5**, p. 53].

THEOREM 1. *If* $M \neq S^2$, *then* C *and* dK *do not vanish at the smooth points of* Z.

Proof. We first show that $Z = \partial\{K > 0\}$. Indeed, if it were not true, there would be an open set $U \subset M$ free of umbilics such that $K \leqq 0$ on U and $U \cap Z \neq \varnothing$. Then $\omega \leqq 0$ with $\omega = 0$ on $U \cap Z$, so by the sinh–Gordon equation, $\Delta\omega \geqq 0$. By the maximum principle for subharmonic functions, ω must vanish, and this implies that $K = 0$ on U; this is a contradiction.

Let $z_0 \in Z$ be a smooth point and a zero of C. By what we have just shown, there is a component D of the set $\{K > 0\}$ such that $z_0 \in \partial D$. By (4), K is superharmonic on

D and, clearly, vanishes on $\partial D \subset Z$. Moreover, $\bar{D} \neq M$, and hence ∂D is the union of finitely many piecewise analytic closed curves, since otherwise K would vanish identically, which is a contradiction.

Now, on Z, (3) reduces to $|dK|^2 = C$ from which $(dK)(z_0) = 0$. That this is impossible is the content of the following lemma whose proof works only for $n = 2$, and is a special case of the well-known strong maximum principle of E. Hopf (cf. [**11**, p. 65]). For completeness, we include a short and simple geometric proof.

LEMMA 1. *Let $D \subset \mathbb{E}^2$ be a domain bounded by piecewise C^1 closed curves. Let u be a superharmonic function on D, continuous on $\bar{D}$, $u \mid \partial D = 0$, and the normal derivative $\partial u/\partial n = 0$ at a point $z_0 \in \partial D$ at which ∂D is C^1. Then $u = 0$ on D.*

Proof. By the minimum principle $u > 0$ on D (or $u \equiv 0$ on D, in which case there is nothing further to prove). Let V be a simply connected neighborhood of z_0 in $\bar{D}$ such that ∂V consists of a C^1 arc $\Gamma = V \cap \partial D$ and a Jordan arc Γ'. Let Γ'' be a subarc properly contained in Γ'. Map V conformally to the unit disk Δ so that Γ'' is mapped onto the upper semicircle of Δ. Now u must have a positive minimum, say δ, on Γ''. Then the harmonic function $\hat{u}$ that takes the value δ on the upper semicircle and 0 on the lower semicircle of Δ is a harmonic minorant of the transformed u which we designate by $\tilde{u}$. Since $\tilde{u}(z_0) = \hat{u}(z_0) = 0$, we have

$$\frac{\partial \tilde{u}}{\partial n}(z_0) \geqq \frac{\partial \hat{u}}{\partial n}(z_0).$$

Thus, it is enough to show that $(\partial \hat{u}/\partial n)(z_0) > 0$. Scaling to $\delta = 1$, $\hat{u}$ can be realized on the upper half-plane as the (harmonic) function $H(z) = \theta/\pi$. Now, $\theta = \arctan(y/x)$, and so $\partial\theta/\partial y = x/(x^2+y^2) > 0$ for $x > 0$.

REMARKS. (a) The converse of Theorem 1 is also true and boils down to the implicit function theorem, namely, *if $z_0 \in Z$ with $C(z_0) \neq 0$, then z_0 is a smooth point of Z.* For, $|dK|^2 = C$ on Z.

(b) Theorem 1 implies that the critical points of K on Z are isolated on Z. In fact, they are also isolated on M. Indeed, if $z_0 \in Z$, $(dK)(z_0) = 0$, were not isolated there would be an analytic arc $\Gamma \subset Zero(dK)$ across z_0. Then $\Gamma \subset Z$, which is a contradiction. In particular, if $M \neq S^2$ smooth points always exist.

THEOREM 2. *The critical points of K at which K does not have local extrema are isolated.*

Proof. First note that the possible values of K at critical points form a finite set. This follows from the analyticity of the set $Zero(dK) \subset M$ and the fact that K is constant on any arc $\Gamma \subset Zero(dK)$. Now, let z_0 be a critical point of K and assume that K does not attain a local extremum at z_0. Since, by Theorem 1, K has isolated critical points on Z, we may assume that $K(z_0) = c \neq 0$.

Case 1, *in which $c > 0$.* Since c is not a local extremum, there exists a component D of $\{K > c\}$ such that $z_0 \in \partial D$. By (4), K is superharmonic on D, so repeating the proof for $K-c$, $(dK)(z_0) = 0$ is possible only at the non-smooth points of ∂D. There are only finitely many non-smooth points of $\{K = c\}$ (by analyticity), so z_0 is isolated.

Case 2, *in which* $c < 0$. Instead of K, consider $-\frac{1}{2}\log(\lambda_2-\lambda_1)$ which corresponds to ω by the local conformal representation F away from umbilics. Now K does not attain a local extremum at z_0, so there exists a component D of $\{K < c\}$ such that $z_0 \in \partial D$. Clearly, $\bar{D}$ does not contain any umbilics, so $-\frac{1}{2}\log(\lambda_2-\lambda_1)$ is defined on $\bar{D}$ and is non-positive. Moreover, by the sinh–Gordon equation, it is subharmonic on D. Hence, we can repeat the proof of Theorem 1 with the appropriate modifications.

THEOREM 3. *If $M \neq S^2$, any non-smooth point $z_0 \in Z$ is a C^1-meeting point of an even number of general folds.*

Proof. K is the ratio of surface elements with respect to the Gauss map $\phi: M \to S^2$. Thus, Z is the set of singular points of ϕ. We can then use the classification of such points given by Wood [**15**]. Using his terminology, the smooth points of Z comprise the good singular points of ϕ by Theorem 1. Now, assume that $z_0 \in Z$ is not a smooth point. Then, ϕ is clearly not degenerate at z_0, and z_0 is not a good point for ϕ. Moreover, z_0 is not a branch point of ϕ. Otherwise, z_0 would be isolated in the set Z so that K would have a local extremum at z_0. This would imply that ω has a local extremum at z_0 with $\omega(z_0) = 0$. Since ω satisfies the sinh–Gordon equation, this contradicts the maximum principle. The result now follows from the classification given in [**15**].

3. *The Laplacian of K^2 near Z*

The following result is a consequence of Propositions 1 and 2 below.

THEOREM 4. *Let M be a closed surface immersed in $\mathbb{E}^3$ with constant mean curvature, and let γ be the function defined by*

$$\gamma(s) = s\int_{\{K^2 \leqq 1/s\}} \Delta K^2, \qquad s > 0, \tag{5}$$

where $\{K^2 \leqq 1/s\}$ is the set of all $x \in M$ such that $K^2(x) \leqq 1/s$. Then, either M is a round sphere (and therefore $\gamma = 0$) or

$$\lim_{s\to\infty} \gamma(s) = \infty, \tag{6}$$

and the derivative of γ is eventually strictly positive.

COROLLARY. *Let M be a closed surface immersed in $\mathbb{E}^3$ with constant mean curvature. Then, if*

$$\int_{\{K^2 \leqq 1/s\}} \Delta K^2 = O\left(\frac{1}{s}\right) \quad \text{for } s \to \infty,$$

M is a round sphere.

If $M \neq S^2$, then by Theorem 1, C does not vanish identically on Z.

PROPOSITION 1. *If C is not identically zero on Z, then*

$$\lim_{s\to\infty} \gamma(s) = \infty.$$

Proof. By (1),

$$\gamma(s) = 2s\int_{\{K^2 \leqq 1/s\}} (1-8K)\,C - 2s\int_{\{K^2 \leqq 1/s\}} (1-4K)\,K^2.$$

The second term approaches zero as $s \to \infty$. As for the first term, for large s,

$$2s\int_{\{K^2 \leqq 1/s\}} (1-8K)\,C \geqq s\int_{\{K^2 \leqq 1/s\}} C = s\int_{\{|K| \leqq 1/\sqrt{s}\}} C.$$

To show that the right-hand side diverges, we first note that, by hypothesis, C does not vanish on an arc $\Gamma \subset Z$. The proposition is then a consequence of the following elementary lemma applied to a tubular neighborhood of Γ in M.

LEMMA 2. *Let u be a C^1 function on* $[0,1]\times[-1,1]$ *with Zero*$(u) = [0,1]\times\{0\}$. *Then, for some $a > 0$,*

$$\text{Area}\,\{|u| \leqq \varepsilon\} \geqq a\cdot\varepsilon$$

uniformly for $\varepsilon \to 0$.

The last statement in Theorem 4 is the content of the following.

PROPOSITION 2. *Given M as in Theorem* 4, *the function γ is eventually non-decreasing. If the derivative of γ vanishes on a divergent sequence $s_n \to \infty$, then M is a round sphere.*

For the proof we shall require the following two lemmas, where M is considered as a compact Riemann surface.

LEMMA 3. *Let τ be an exhaustion function on $M\backslash Z$. Then, for r large,*

$$\int_{\{\tau \leqq r\}} \Delta \log K^2 = \frac{d}{dr}\int_{\{\tau = r\}} \log K^2 \cdot d^c\tau, \tag{7}$$

where $d^c = i(\bar\partial - \partial)$ (see [3, p. 18] *for notation).*

Proof. For any scalar ρ on $M\backslash Z$,

$$d\tau \wedge d^c\rho - d\rho \wedge d^c\tau = 2i(\partial\rho \wedge \partial\tau - \bar\partial\rho \wedge \bar\partial\tau) = 0.$$

Since $d\rho \wedge d^c\tau = (\partial\rho/\partial\tau)\,d\tau \wedge d^c\tau$, we obtain

$$d^c\rho \equiv \frac{\partial\rho}{\partial\tau}\,d^c\tau \quad (\text{mod}\, d\tau).$$

Putting $\rho = \log K^2$,

$$\int_{\{\tau \leqq r\}} \Delta \log K^2 = \int_{\{\tau \leqq r\}} dd^c \log K^2 = \int_{\{\tau = r\}} d^c \log K^2 = \int_{\{\tau = r\}} \frac{\partial \log K^2}{\partial\tau}\,d^c\tau,$$

where the last equality is a consequence of the fact that $d\tau = 0$ along $\{\tau = r\}$. Now, since the integral and $\partial/\partial\tau$ can be interchanged (cf. [3, pp. 86–87]), formula (7) follows.

LEMMA 4. *The function K^2 has no critical points in $M\backslash Z$ near Z.*

Proof. The set $Zero(d(K^2))$ is analytic. It is, therefore, the union of finitely many analytic curves. If $\Gamma \subset Zero(d(K^2))$ is an analytic arc and if $\Gamma \cap Z \neq \emptyset$, then $\Gamma \subset Z$ since K^2 is constant along Γ.

Proof of Proposition 2. If $M \neq S^2$, then by Lemma 4, we can choose

$$\tau = \log\log(1/K^2)$$

near Z. We may then extend it to an exhaustion function τ on the whole of $M \backslash Z$. Along $\{\tau = r\}$, for r large,

$$d^c\tau = d^c\left(\log\log\frac{1}{K^2}\right) = \frac{1}{\log K^2}\frac{d^cK^2}{K^2},$$

and so (7) becomes

$$\int_{\{\tau \leqq r\}} \Delta \log K^2 = \frac{d}{dr}\left\{e^{e^r}\int_{\{\tau = r\}} d^cK^2\right\}.$$

By a change of variable $s = e^{e^r}$ we obtain, for s large,

$$\begin{aligned}\frac{1}{s\log s}\int_{\{K^2 \geqq 1/s\}} \Delta \log K^2 &= \frac{d}{ds}\left\{s\int_{\{K^2 = 1/s\}} d^cK^2\right\}\\ &= -\frac{d}{ds}\left\{s\int_{\{K^2 \leqq 1/s\}} \Delta K^2\right\} = -\gamma'(s),\end{aligned} \tag{8}$$

where in the second equality Stokes's theorem is used. Now, by the superharmonicity of $\log K^2$, γ is eventually non-decreasing. Finally, if $\gamma'(s_n) = 0$ for some sequence $s_n \to \infty$ then, again by the superharmonicity of $\log K^2$ and (8), it follows that $\Delta \log K^2 = 0$ on $M \backslash Z$; this is a contradiction.

REMARKS. (a) If $M \neq S^2$, then by l'Hospital's rule

$$\lim_{s\to\infty}\left(\int_{\{K^2 \leqq 1/s\}} \Delta K^2\right) > 0.$$

However,

$$\lim_{s\to\infty}\frac{d}{ds}\int_{\{K^2 \leqq 1/s\}} \Delta K^2 = 0.$$

(b) The possibility of immersing closed surfaces of genus 2 into $\mathbb{E}^3$ with constant mean curvature remains unsolved. One of the principal difficulties here, as was already pointed out by Hopf, is the presence of umbilical points at which the sinh–Gordon equation becomes singular. (This does not occur in Wente's example.) For $g = 2$, the analytic behavior of the Gaussian curvature near its critical points may give rise to a Morse type topological restriction for the immersion to exist. The reason that the construction due to Kapouleas fails for $g = 2$ is that the required balancing condition is violated in this case.

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Department of Mathematics
University of Illinois
1409 West Green Street
Urbana
Illinois 61801
USA

Department of Mathematics and Statistics
Queen's University
Kingston
Ontario
Canada K7L 3N6

Department of Mathematics
Rutgers University
Camden
New Jersey 08102
USA