Crowd dynamics

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Group of intelligent agents on the move
Autonomous, Self-propelled, Self-driven, Selfish, Greedy, Boids,!
Conservation laws on networks

1. The only conservation at nodes does not determine the dynamics
2. Additional rules should take into account, as distribution policies
3. Solutions give rise to boundary value problems on arcs
4. Entropy is used to determine dynamics (maximal flux)
Dynamics at junctions

Rule (A) : Out. Fluxes Vector = A " Inc. Fluxes Vector

Traffic distribution matrix A = (# ji ), 0<# ji<1, $j # ji =1

Rule (B) : Max %Inc. Fluxes Vector%

Rule (B) is an “entropy” type rule : maximize velocity
Berkeley-Nokia and Octotelematics

MOBILE CENTURY – Using GPS Mobile Phones as Traffic Sensors

Daniel Work, Olli-Pekka Tossavainen, Alexandre Bayen
Systems Engineering, UC Berkeley

OCTO Telematics 2008

The “Clear Box”

GSM/GPRS
Supports the transmission of the data from the “Clear Box” to the Service Centre as well as the software update data over the air.

GPS
Allows to locate the vehicle through the GPS coordinates (latitude and longitude) and projected on digital maps.

ACCELERATION SENSOR
Allows crash detection and dynamics reconstruction before and after the crash event.

ANCILLARY CIRCUITS
Self-check and diagnostics; continuous operation; integrated back-up battery

AUTOMOTIVE
Certified for compliance with the automotive standards; it is approved by TÜV according to CE norms and ISO 7637 specifications.

INSTALLATION
It’s easy, quick and not invasive of the interior design and of the on-board technology. 3 to 6 contact points with the car’s electrical system, depending on the service required.

OPTIONS
Options include panic button for emergency (e-call), panic free, CAN interface for the GEOSS application, Engine Stop at 0 km/h Speed.

APPLICATION SOFTWARE
Allows managing, processing, recording, and transmission of data. Furthermore, performs self-checking and diagnostics functions.

Berkeley-Nokia: Alex Bayen group, Octotelematics: Corrado DeFabritiis
Tens, hundreds, thousands of pedestrians

Helbing et al., microscopic

\[ \frac{dv_{\alpha}}{dt} = \frac{v_{\alpha} e_{\alpha} - v_{\alpha}}{\tau_{\alpha}} - \sum_{\beta} \nabla V_{\alpha \beta}[b(r_{\alpha \beta})] \]

Maury-Venel, microscopic

\[ q(t) = q_0 + \int_{0}^{t} P_{C_q} U(q(s)) \, ds \]

Colombo-Rosini, macroscopic 1D

\[ \partial_t \rho + \partial_x q(\rho) = 0 \]

Bellomo-Dogbé, macroscopic

\[ \begin{cases} 
\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \\
\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = F[\rho, \mathbf{v}] 
\end{cases} \]
Time evolving measures

Measure $\mu: (t,E) \to \mu(t,E)$ number of pedestrians in the region $E$

Flow map $\gamma: x \to x + \nu(x,\mu) \Delta t$ move points with given velocity

At next time step is given by $\mu(t+\Delta t, E) = \mu(t, \gamma^{-1}(E))$

The velocity $\nu$ is the sum of desired velocity $\nu_d$ and interaction term $\nu_i(\mu)$

Time evolving measures: Canuto-Fagnani-Tilli, Tosin-P., Muntean et al., Santambrogio, Carrillo-Figalli et al., Colombo, Gwiazda !
Macroscopic for self-organization in pedestrians

**Initial condition**

**Desired velocity field**

Exiting the metro: simulation

**MACRO**

**MICRO**

**MULTISCALE**

**MACRO**
What metric for evolving measures?

If \( \# < \bar{\theta} \) then the L1 distance is \( O(1) \).
Wasserstein (Vaserstein) metric

$(X, d)$ metric space

$\mu_1, \mu_2$ probability measures

$T \# \mu_1 = \mu_2$

$\inf_{T \# \mu_1 = \mu_2} \int_X d(x, T(x)) \, d\mu_1$

$v[\mu]$ is uniformly Lipschitz and uniformly bounded, i.e. there exist $L, M$ not depending on $\mu$, such that for all $\mu \in \mathcal{M}, x, y \in \mathbb{R}^n$,

$$|v[\mu](x) - v[\mu](y)| \leq L|x - y| \quad |v[\mu](x)| \leq M.$$ 

$v$ is a Lipschitz function, i.e. there exists $K$ such that

$$\|v[\mu] - v[\nu]\|_{C^0} \leq KW_p(\mu, \nu).$$
Generalized Wasserstein

\[
\begin{align*}
\partial_t \mu + \nabla \cdot (\nu [\mu] \mu) &= h [\mu], \\
\mu|_{t=0} &= \mu_0.
\end{align*}
\]

\[
W_{p}^{a,b} (\mu, \nu) = \inf_{\tilde{\mu}, \tilde{\nu} \in \mathcal{M}^p, |\tilde{\mu}| = |\tilde{\nu}|} (a |\mu - \tilde{\mu}| + a |\nu - \tilde{\nu}| + b W_{p}(\tilde{\mu}, \tilde{\nu}))
\]

\[
\mathcal{B}^{a,b} [\mu, \nu, h] := a^2 \left( \int_0^1 dt \left( \int_{\mathbb{R}^d} d|h_t| \right) \right)^2 + b^2 \int_0^1 dt \left( \int_{\mathbb{R}^d} d\mu_t |v_t|^2 \right).
\]

\[
T_{2}^{a,b} (\mu, \nu) = \inf_{\tilde{\mu}, \tilde{\nu} \in \mathcal{M}, |\tilde{\mu}| = |\tilde{\nu}|} a^2 (|\mu - \tilde{\mu}| + |\nu - \tilde{\nu}|)^2 + b^2 W_{2}^2(\tilde{\mu}, \tilde{\nu})
\]

\[
T_{2}^{a,b} (\mu_0, \mu_1) = \inf \{ \mathcal{B}^{a,b} [\mu, \nu, h] \mid \mu \text{ is a solution of (1)} \}
\]

with vector field \(\nu\), source \(h\) and \(\mu|_{t=0} = \mu_0, \mu|_{t=1} = \mu_1\)