

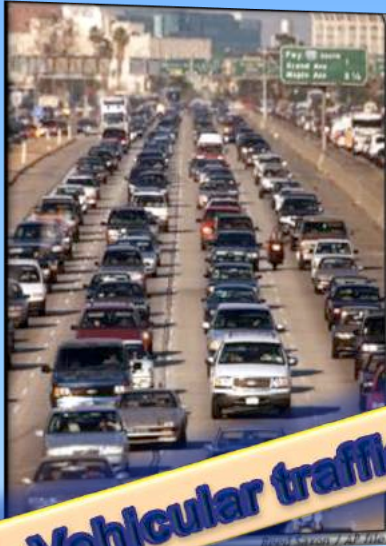
# Crowd dynamics

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# Group of intelligent agents on the move

Autonomous, Self-propelled, Self-driven, Selfish, Greedy, Boids, !



**Vehicular traffic**



**Crowd dynamics**

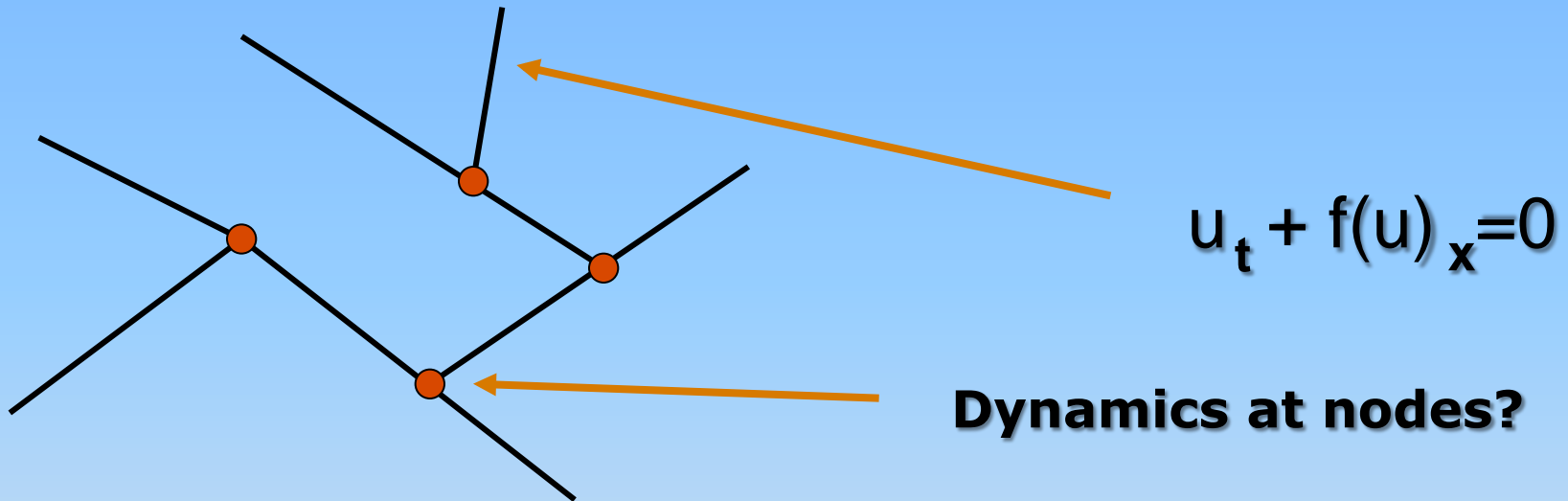


**Networked robots**

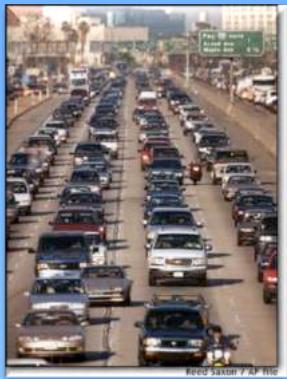


**Animal groups**

# Conservation laws on networks



1. The only conservation at nodes does not determine the dynamics
2. Additional rules should take into account, as distribution policies
3. Solutions give rise to boundary value problems on arcs
4. Entropy is used to determine dynamics (maximal flux)



Vehicular Traffic

## Transportation

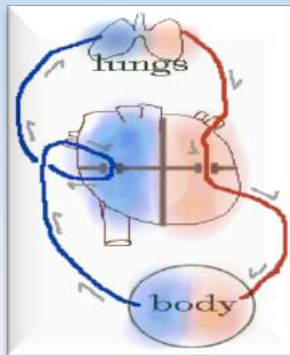


Irrigation Channels

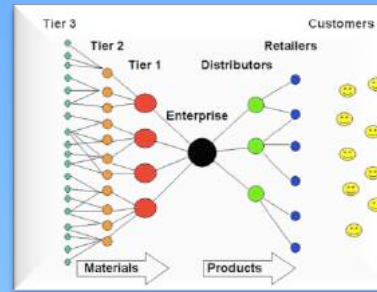


Gas pipelines

## Real fluids

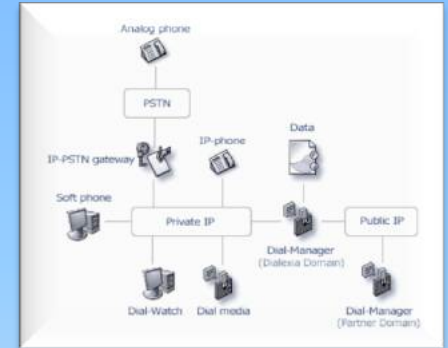


Blood circulation



Supply chains

## Services/Supply

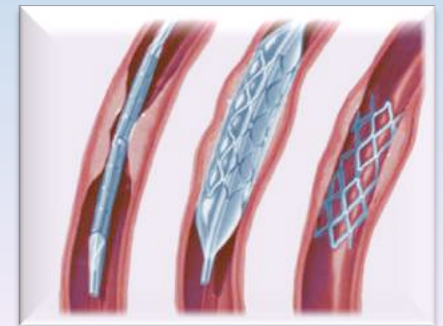


Tlc and data networks



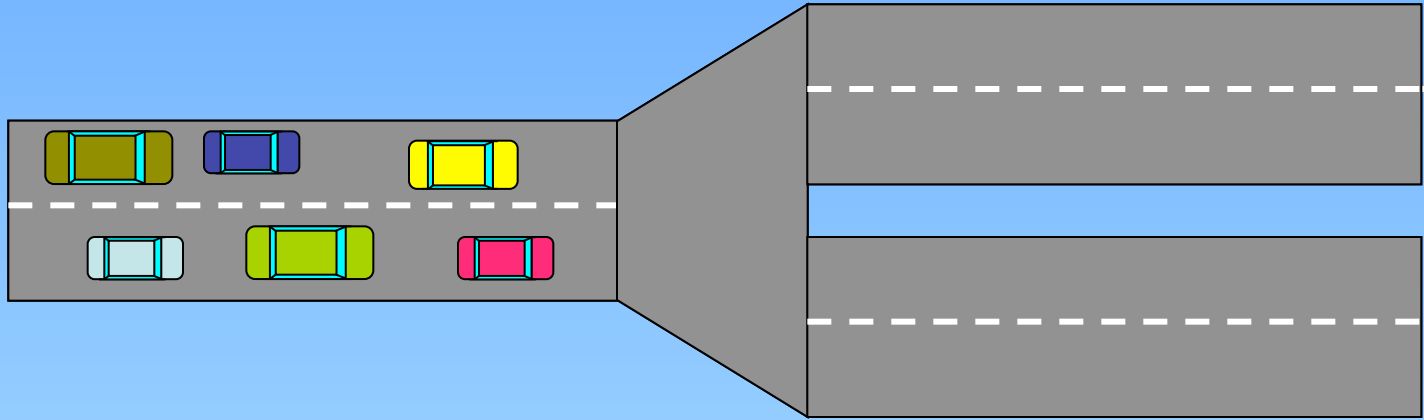
Air traffic management

## Bio-Medical



Vascular stents

# Dynamics at junctions



Rule (A) : Out. Fluxes Vector =  $A$  " Inc. Fluxes Vector

Traffic distribution matrix  $A = (\#_{ji})$  ,  $0 < \#_{ji} < 1$  ,  $\sum_j \#_{ji} = 1$

Rule (B) : Max  $\sum_j \#_{ji}$  Inc. Fluxes Vector

Rule (B) is an “entropy” type rule : maximize velocity



# Berkeley-Nokia and Octotelematics

## MOBILE CENTURY – Using GPS Mobile Phones as Traffic Sensors



## OCTO Telematics 2008

## The "Clear Box"

### GSM/GPRS

Supports the transmission of the data from the "Clear Box" to the Service Centre, as well as the software up-date over the air.

### INSTALLATION

It's easy, quick and not invasive of the internal design and of the on-board technology. 3 to 6 contacts points with the car's electrical system, depending on the services required.

### GPS

Allows to localise the vehicle through the GPS coordinates (latitude and longitude) and projected on digital maps.

### AUTOMOTIVE

Certified for compliance with the automotive standards, it's approved by TUV according to CE norms and ISO 7637 specifications.

### ACCELERATION SENSOR

Allows crash detection and dynamics reconstruction before and after the crash event.

### OPTIONS

Options include: panic button for emergency (e-call), hands free, CAN interface for the OE/OES application. Engine Stop at 0 km Speed.

### ANCILLARY CIRCUITS

Self-check and diagnostics: continuous operations. Integrated back-up battery

### APPLICATION SOFTWARE

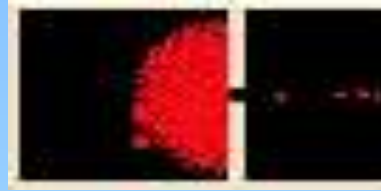
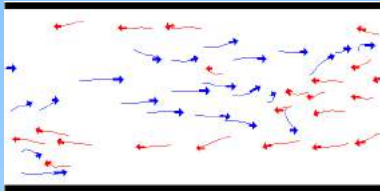
Allows managing, processing, recording, and transmission of data. Furthermore, performs self-checking and diagnostics functions.



Berkeley-Nokia: Alex Bayen group, Octotelematics: Corrado DeFabritiis

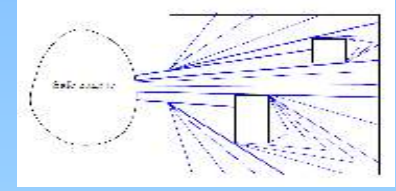
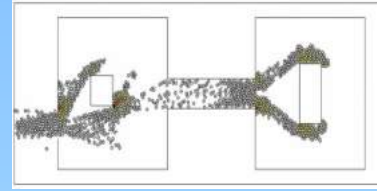
# Tens, hundreds, thousands of pedestrians

Helbing et al., microscopic



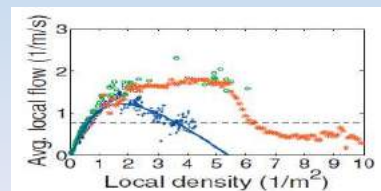
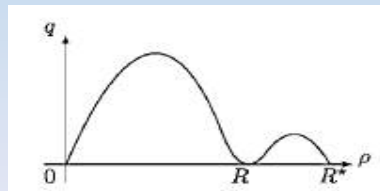
$$\frac{dv_{\alpha}}{dt} = \frac{v_{\alpha}^0 e_{\alpha} - v_{\alpha}}{\tau_{\alpha}} - \sum_{\beta} \nabla V_{\alpha\beta} [b(r_{\alpha\beta})]$$

Maury-Venel, microscopic



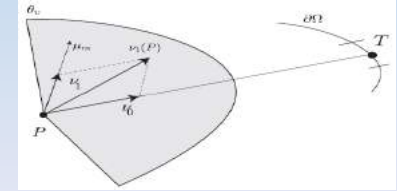
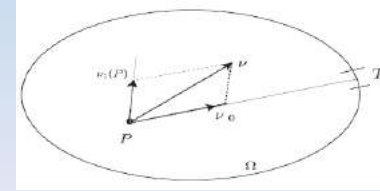
$$q(t) = q_0 + \int_0^t P_{C_q} U(q(s)) ds$$

Colombo-Rosini, macroscopic 1D



$$\partial_t \rho + \partial_x q(\rho) = 0$$

Bellomo-Dogbé, macroscopic



$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho v) = 0 \\ \partial_t v + (v \cdot \nabla) v = F[\rho, v] \end{cases}$$

# Time evolving measures

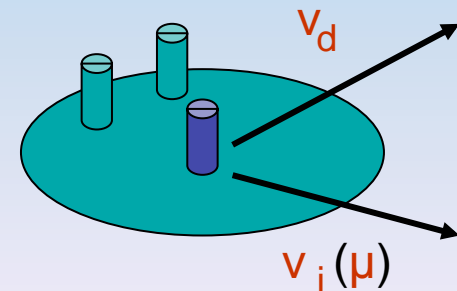
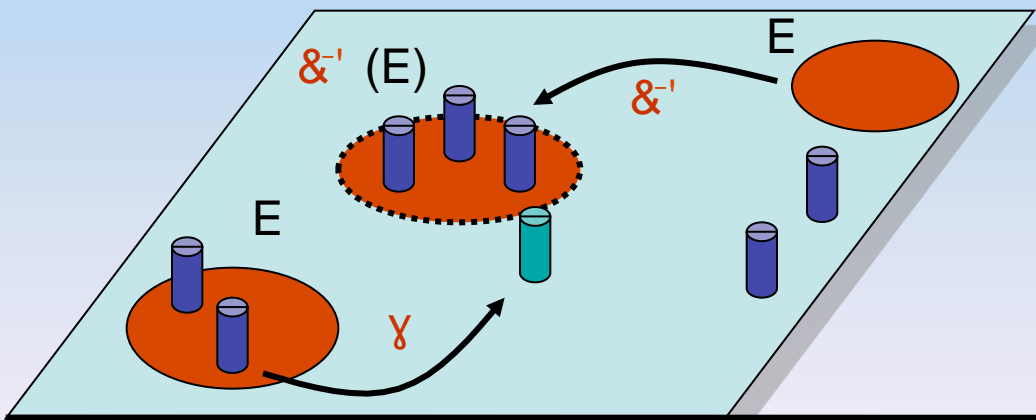
Measure  $\mu: (t, E) \rightarrow \mu(t, E)$  number of pedestrians in the region  $E$

Flow map  $\gamma: x \rightarrow x + v(x, \mu) \Delta t$  move points with given velocity

At next time step is given by  $\mu(t + \Delta t, E) = \mu(t, \gamma^{-1}(E))$

The velocity  $v$  is the sum of desired velocity  $v_d$

and interaction term  $v_i(\mu)$

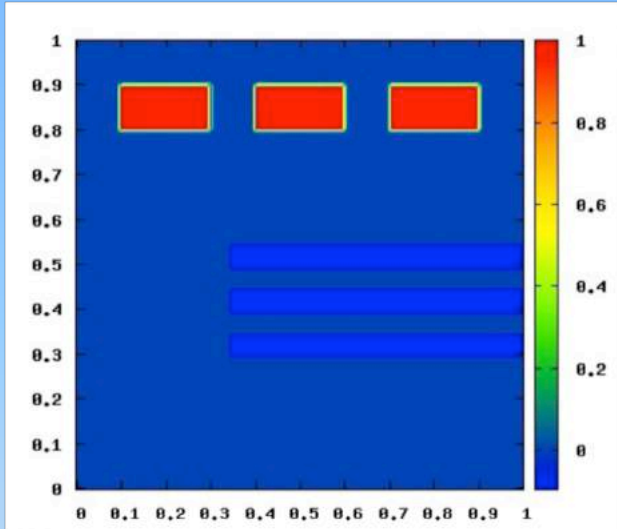


Time evolving measures: Canuto-Fagnani-Tilli, Tosin-P., Muntean et al., Santambrogio, Carrillo-Figalli et al., Colombo, Gwiazda !.

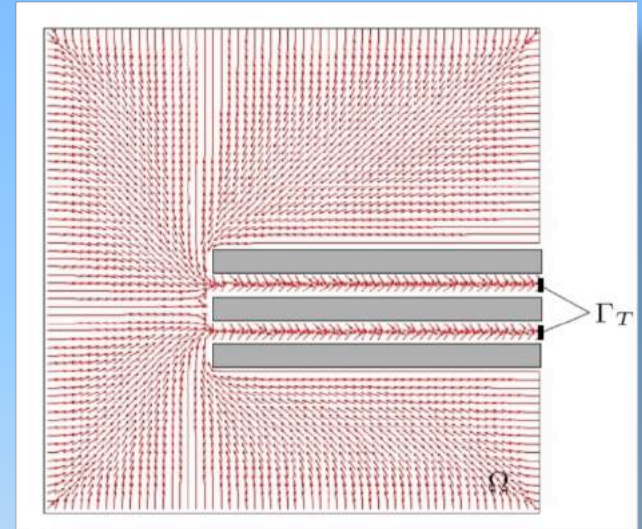


# Macroscopic for self-organization in pedestrians

Initial condition



Desired velocity field



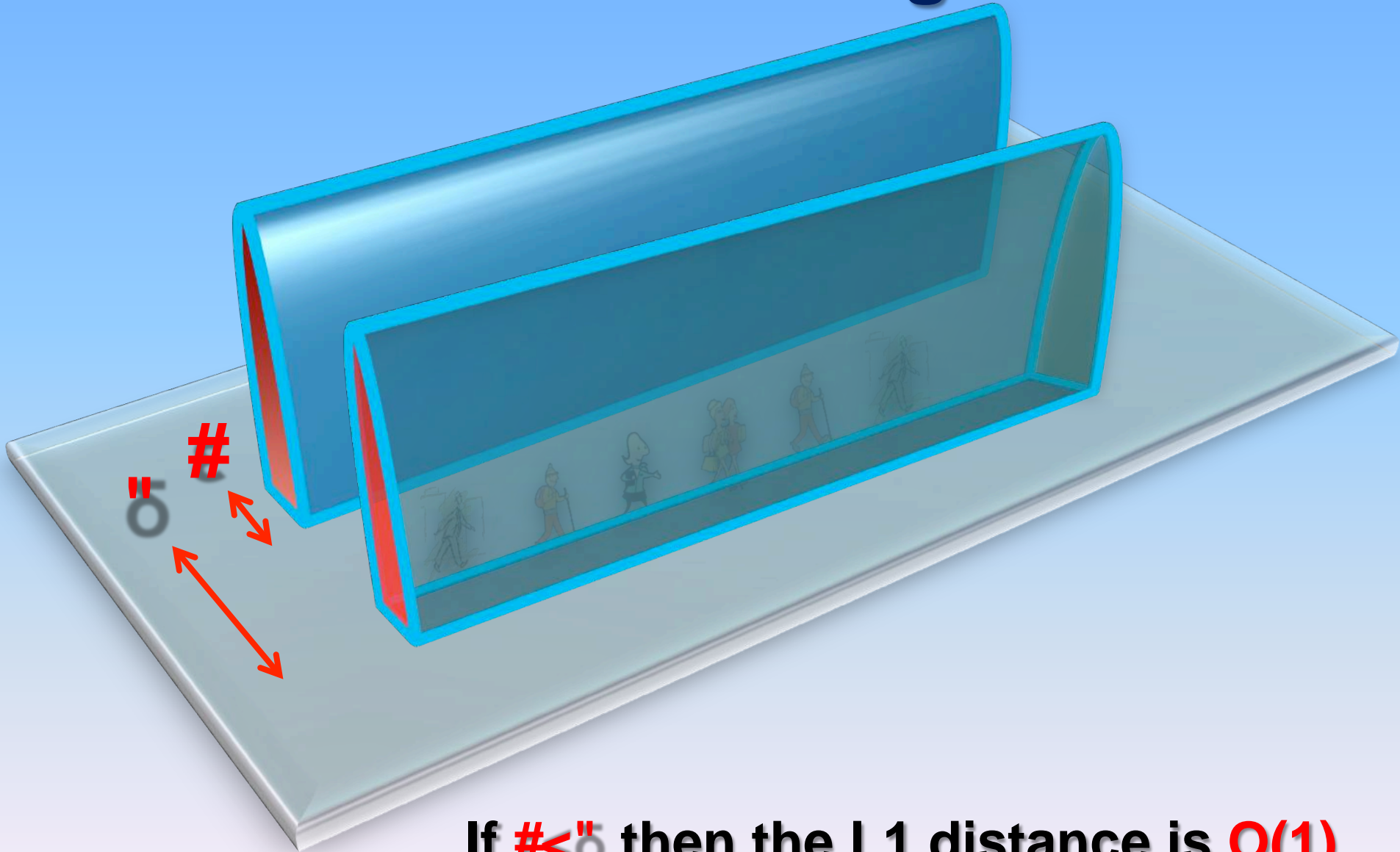
## Exiting the metro: simulation



MICRO  
MULTISCALE  
MACRO



# What metric for evolving measures?



If  $\# < "$  then the L1 distance is  $O(1)$

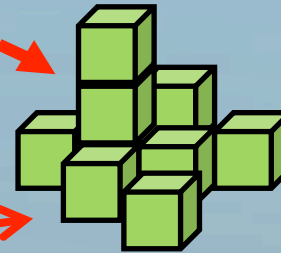
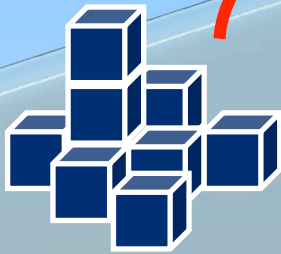
# Wasserstein (Vaserstein) metric

$(X, d)$  metric space

$\mu_1, \mu_2$  probability measures

$$T\#\mu_1 = \mu_2$$

$$\inf_{T\#\mu_1 = \mu_2} \int_X d(x, T(x)) d\mu_1$$



$d$

$$\begin{cases} \partial_t \mu + \nabla \cdot (v\mu) = 0 \\ \mu|_{t=0} = \mu_0 \end{cases}$$

$v[\mu]$  is uniformly Lipschitz and uniformly bounded, i.e. there exist  $L, M$  not depending on  $\mu$ , such that for all  $\mu \in \mathcal{M}, x, y \in \mathbb{R}^n$ ,

$$|v[\mu](x) - v[\mu](y)| \leq L|x - y| \quad |v[\mu](x)| \leq M.$$

$v$  is a Lipschitz function, i.e. there exists  $K$  such that

$$\|v[\mu] - v[\nu]\|_{C^0} \leq KW_p(\mu, \nu).$$

# Generalized Wasserstein

$$\begin{cases} \partial_t \mu + \nabla \cdot (v [\mu] \mu) = h [\mu] , \\ \mu|_{t=0} = \mu_0 . \end{cases}$$

$$W_p^{a,b}(\mu, \nu) = \inf_{\substack{\tilde{\mu}, \tilde{\nu} \in \mathcal{M}^p \\ |\tilde{\mu}| = |\tilde{\nu}|}} (a|\mu - \tilde{\mu}| + a|\nu - \tilde{\nu}| + bW_p(\tilde{\mu}, \tilde{\nu}))$$

$$\mathcal{B}^{a,b}[\mu, v, h] := a^2 \left( \int_0^1 dt \left( \int_{\mathbb{R}^d} d|h_t| \right) \right)^2 + b^2 \int_0^1 dt \left( \int_{\mathbb{R}^d} d\mu_t |v_t|^2 \right) .$$

$$T_2^{a,b}(\mu, \nu) = \inf_{\tilde{\mu}, \tilde{\nu} \in \mathcal{M}, |\tilde{\mu}| = |\tilde{\nu}|} a^2 (|\mu - \tilde{\mu}| + |\nu - \tilde{\nu}|)^2 + b^2 W_2^2(\tilde{\mu}, \tilde{\nu})$$

$$T_2^{a,b}(\mu_0, \mu_1) = \inf \{ \mathcal{B}^{a,b}[\mu, v, h] \mid \mu \text{ is a solution of (1)} \}$$

with vector field  $v$ , source  $h$  and  $\mu|_{t=0} = \mu_0, \mu|_{t=1} = \mu_1$



# CROWD DYNAMICS

# VEHICULAR TRAFFIC

# SUPPLY CHAINS

## SOCIAL



Francesco Rossi



Anna Chiara Lai



Marco Caponigro



Paolo Frasca



Massimo Fornasier



Emmanuel Trelat



Andrea Tosin



Emiliano Cristiani



Yacine Chitour



Paola Goatin



Roberto Natalini



Alex Bayen



Rinaldo Colombo



Mauro Garavello



Dirk Helbing



Corrado Lattanzio



Giuseppe Coclite



Alessia Marigo



Dan Work



Seb Blandin



Amelio Maurizi



Simone Goettlich



Gabriella Bretti



Ciro D'Apice



Rosanna Manzo



Axel Klar



Michael Herty

## ANIMAL GROUPS