Crowd dynamics

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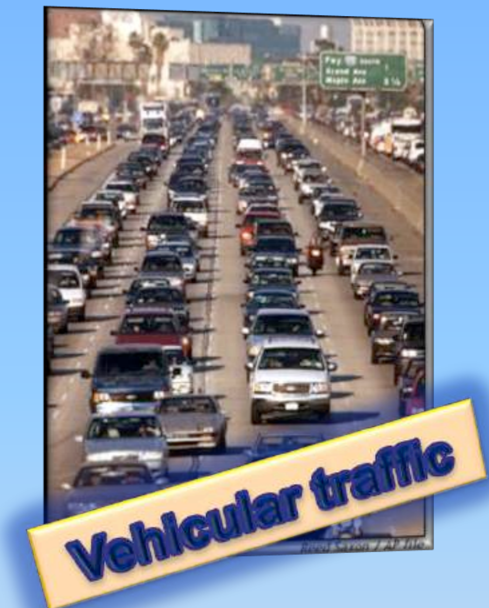
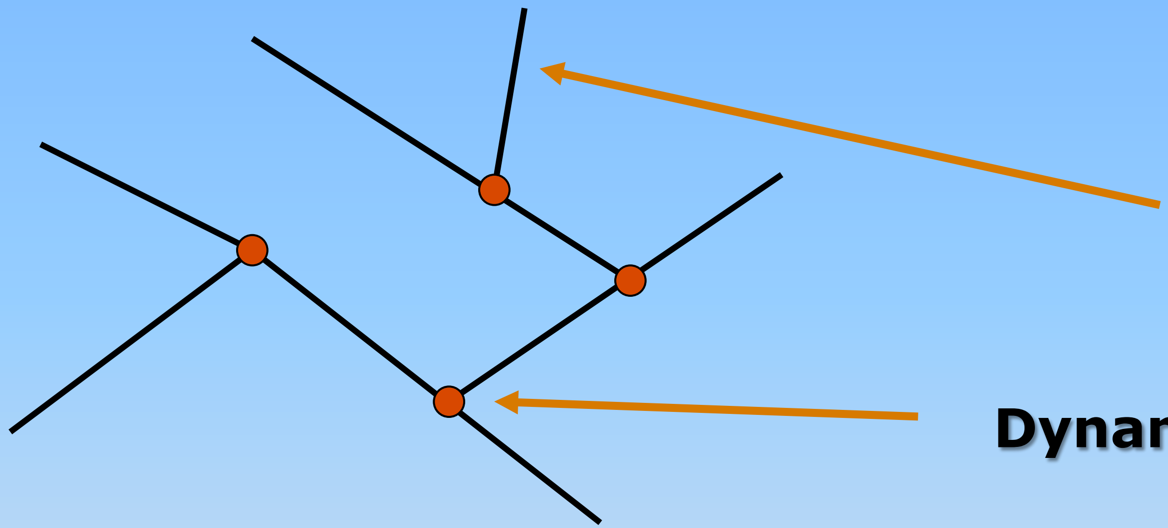
Group of intelligent agents on the move

Autonomous, Self-propelled, Self-driven, Selfish, Greedy, Boids, !



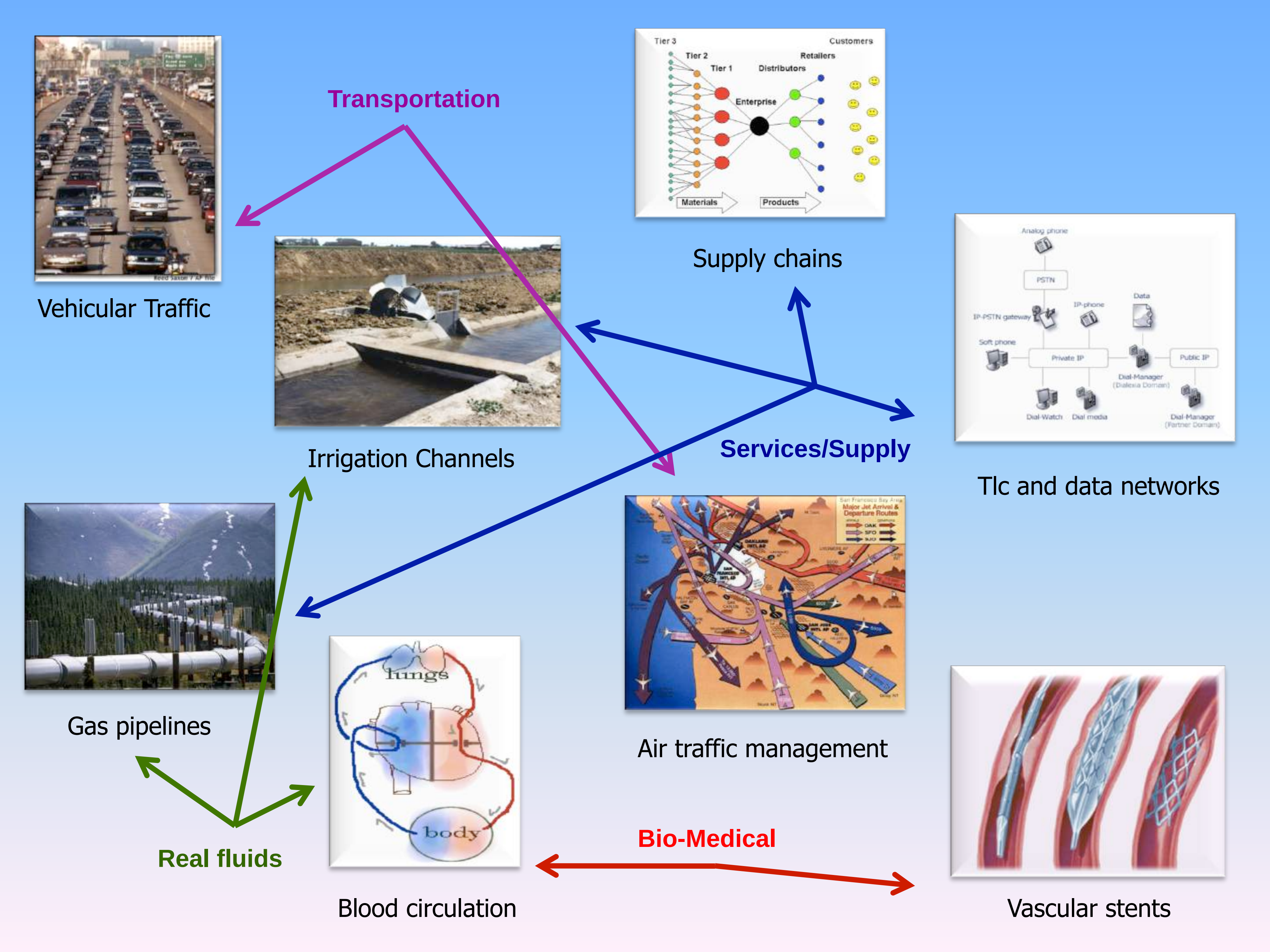
Conservation laws on networks



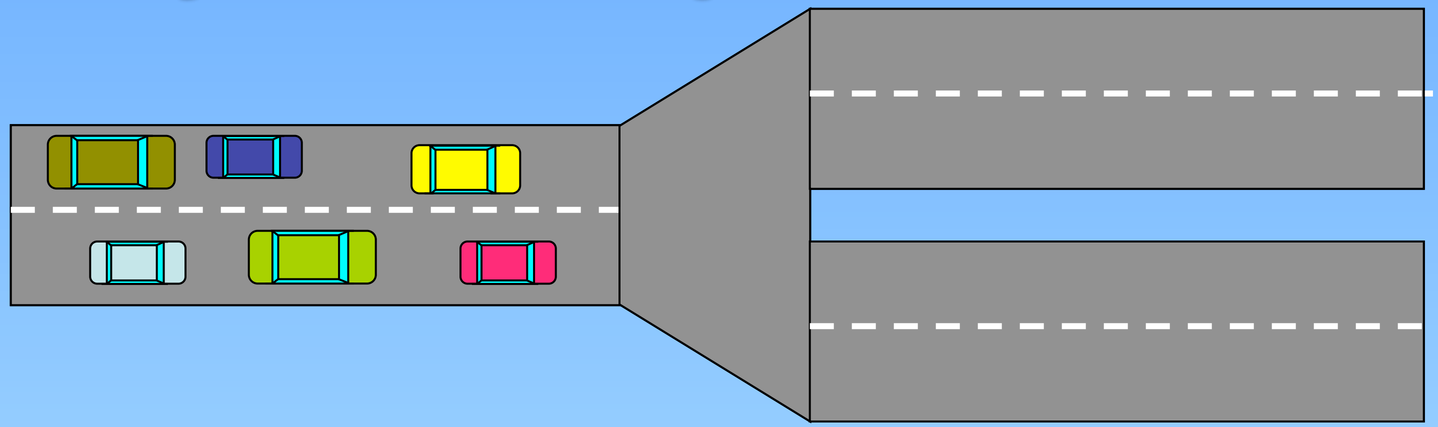
$u_t + f(u)_x = 0$

Dynamics at nodes?

1. **The only conservation at nodes does not determine the dynamics**
2. **Additional rules should take into account, as distribution policies**
3. **Solutions give rise to boundary value problems on arcs**
4. **Entropy is used to determine dynamics (maximal flux)**


Transportation
Vehicular Traffic
Irrigation Channels
Tier 3
Customers
Tier 2
Retailers
Tier 1
Distributors
Enterprise
Materials
Products
Supply chains
Analog phone
PSTN
Data
IP-PSTN gateway
IP-phone
Soft phone
Private IP
Public IP
Dial-Manager
(Dialexia Domain)
Dial-Watch
Dial media
Dial-Manager
(Partner Domain)
Services/Supply
Tlc and data networks
Gas pipelines
Major Jet Arrival & Departure Routes
OAK
SFO
SJC
Air traffic management
lungs
body
Real fluids
Bio-Medical
Blood circulation
Vascular stents

Dynamics at junctions



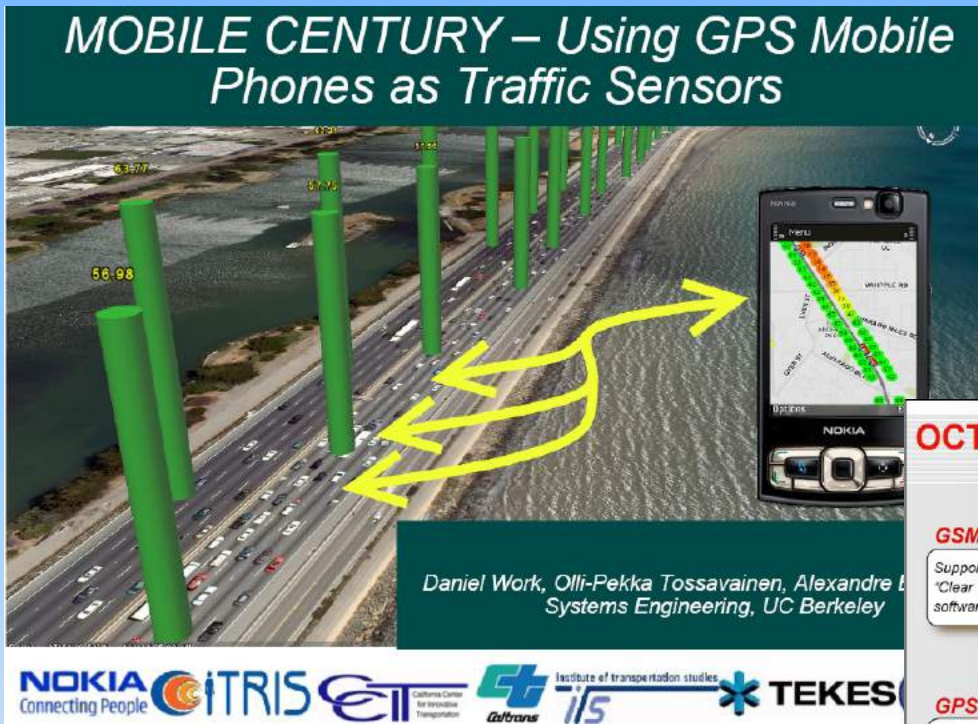
Rule (A) : Out. Fluxes Vector = A " Inc. Fluxes Vector

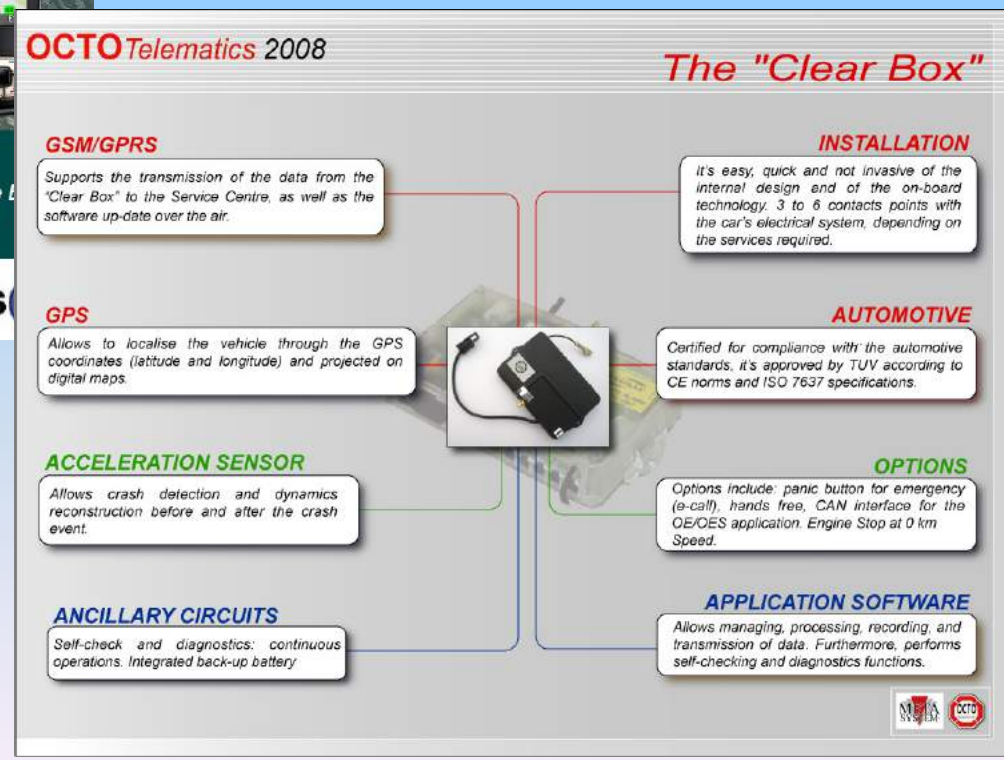
Traffic distribution matrix A = (#$_{ji}$) , 0<#$_{ji}$<1, $ $_{j}$ #$_{ji}$ =1

Rule (B) : Max %Inc. Fluxes Vector%

Rule (B) is an “entropy” type rule : maximize velocity

Berkeley-Nokia and Octotelematics

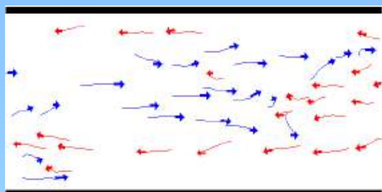


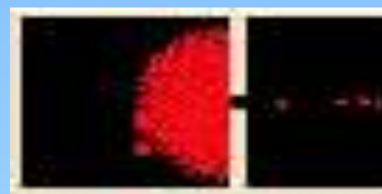


Berkeley-Nokia: Alex Bayen group, Octotelematics: Corrado DeFabritiis

Tens, hundreds, thousands of pedestrians

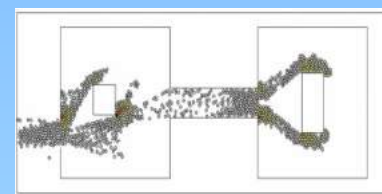
Helbing et al., microscopic

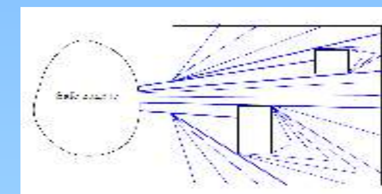




$$\frac{dv_\alpha}{dt} = \frac{v_\alpha^0 e_\alpha - v_\alpha}{\tau_\alpha} - \sum_\beta \nabla V_{\alpha\beta}[b(r_{\alpha\beta})]$$

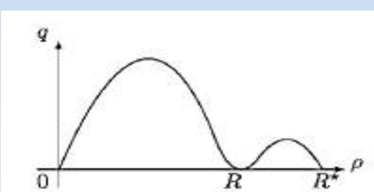
Maury-Venel, microscopic

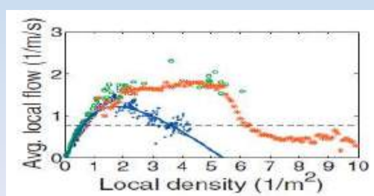




$$q(t) = q_0 + \int_0^t P_{C_q} U(q(s))\, ds$$

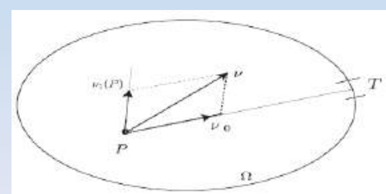
Colombo-Rosini, macroscopic 1D

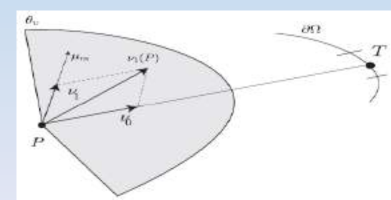




$$\partial_t \rho + \partial_x q(\rho) = 0$$

Bellomo-Dogbé, macroscopic





$$\begin{cases} \partial_t \rho + \nabla \cdot (\rho v) = 0 \\ \partial_t v + (v \cdot \nabla) v = F[\rho,\, v] \end{cases}$$

Time evolving measures

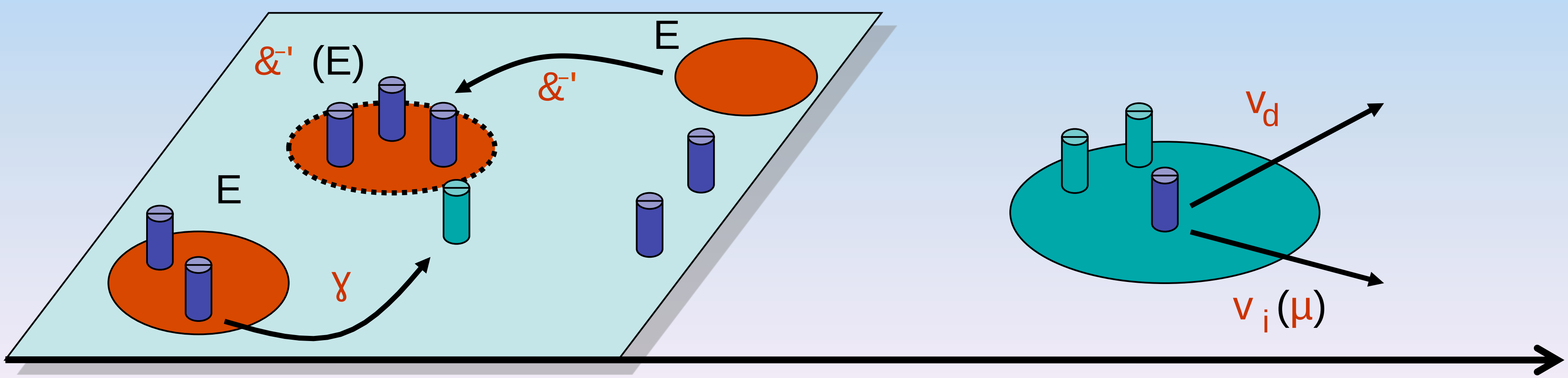
Measure μ: (t,E) → μ(t,E) number of pedestrians in the region E

Flow map ɣ: x → x + v(x,μ) Δt move points with given velocity

At next time step is given by μ(t+!Δt ,E) = μ(t,$ɣ^{-1}$ (E))

The velocity v is the sum of desired velocity v_d

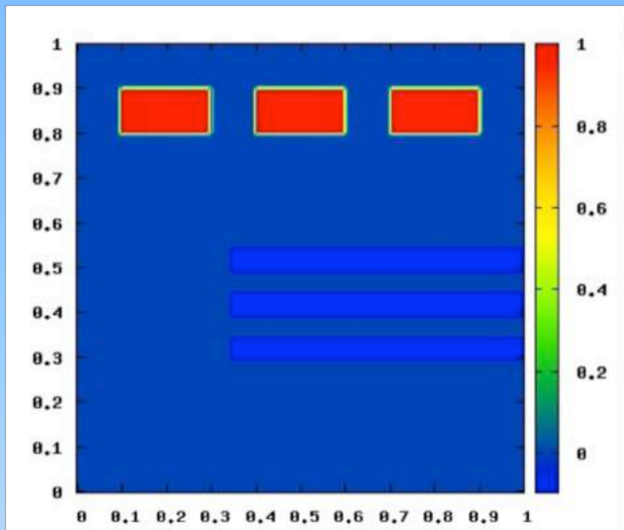
and interaction term v_i (μ)



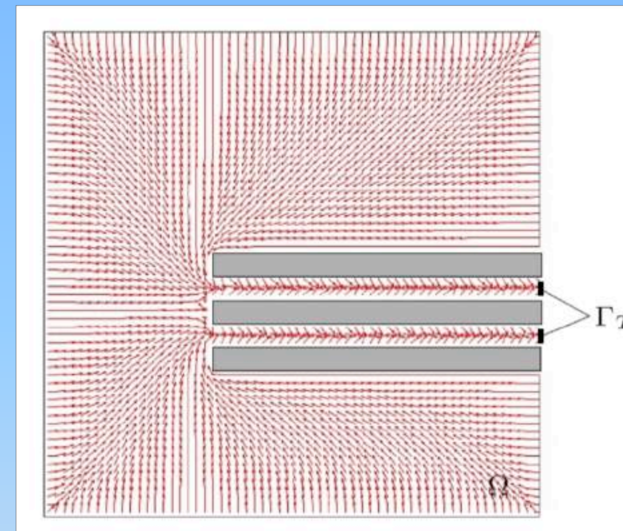
Time evolving measares: Canuto-Fagnani-Tilli, Tosin-P., Muntean et al., Santambrogio, Carrillo-Figalli et al., Colombo, Gwiazda !.

Macroscopic for self-organization in pedestrians

Initial condition



Desired velocity field



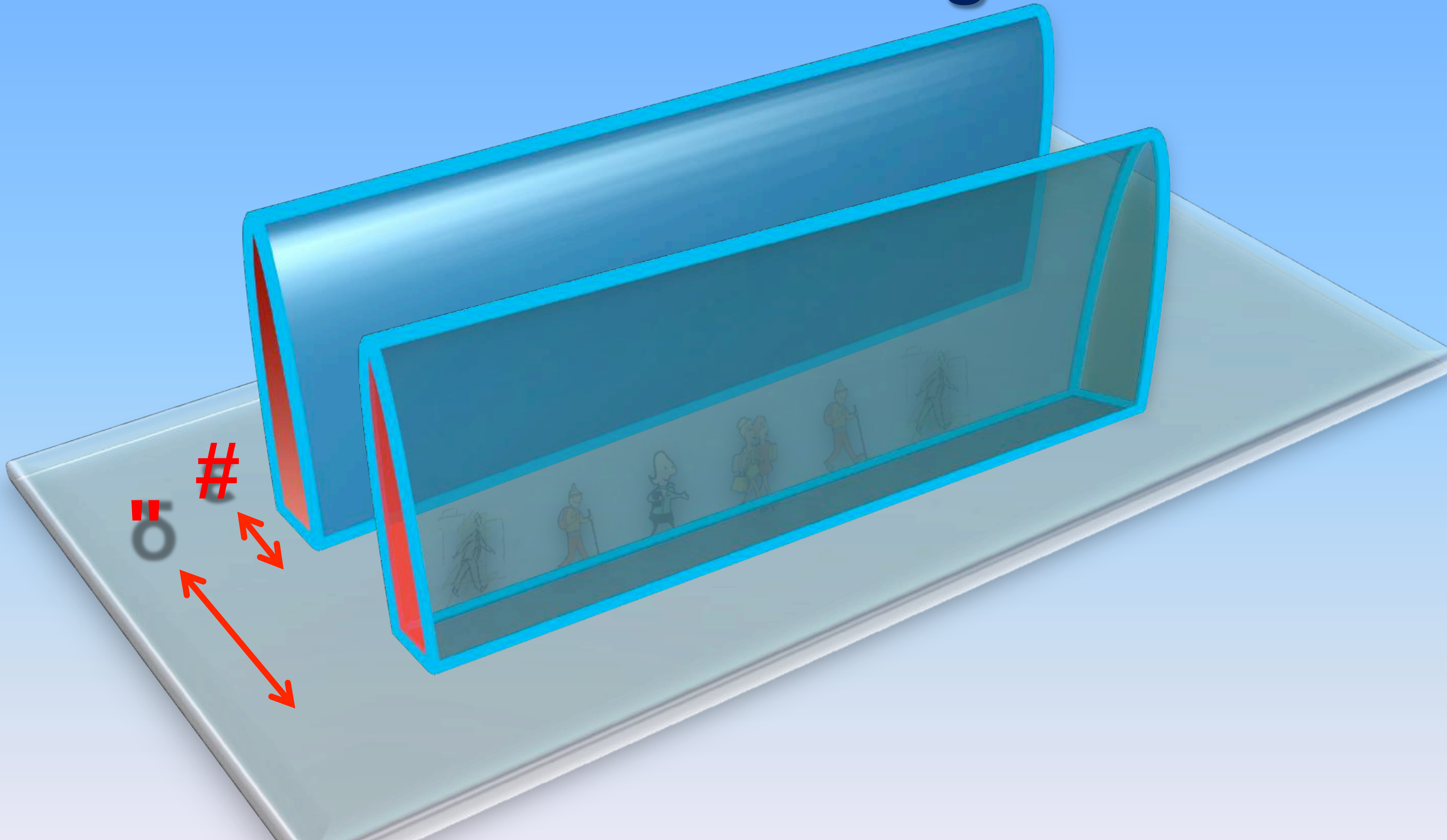
Exiting the metro: simulation

MICRO

<u>MULTISCALE</u>

MACRO

What metric for evolving measures?



If #<ő then the L1 distance is O(1)

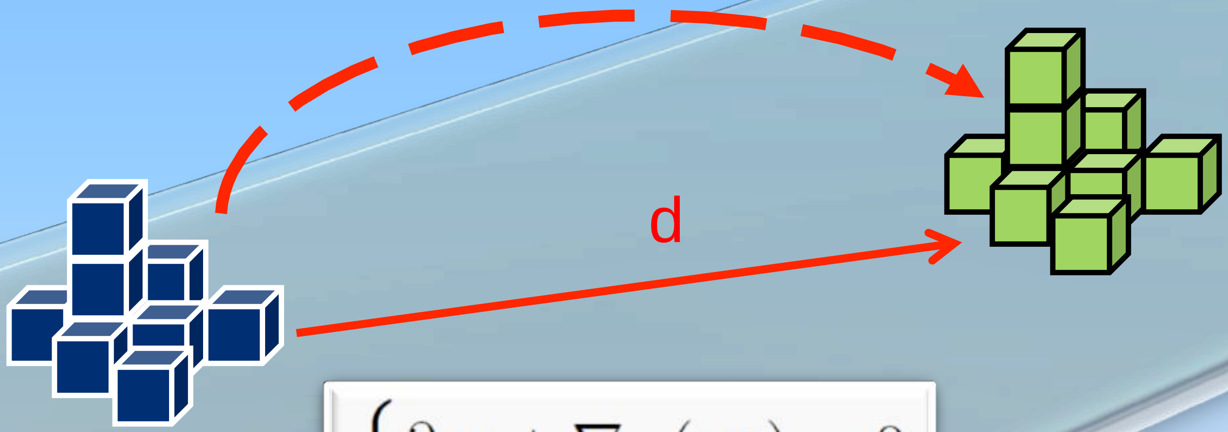
Wasserstein (Vaserstein) metric

(X, d) metric space

$\mu_1, \ \mu_2$ probability measures

$T\#\mu_1 = \mu_2$

$$\inf_{T\#\mu_1=\mu_2} \int_X d(x, T(x))\, d\mu_1$$



$$\begin{cases} \partial_t \mu + \nabla \cdot (v\mu) = 0 \\ \mu_{|t=0} = \mu_0 \end{cases}$$

$v[\mu]$ is uniformly Lipschitz and uniformly bounded, i.e. there exist L, M not depending on μ, such that for all $\mu \in \mathcal{M}, x, y \in \mathbb{R}^n$,

$$|v[\mu](x) - v[\mu](y)| \leq L|x - y| \qquad |v[\mu](x)| \leq M.$$

v is a Lipschitz function, i.e. there exists K such that

$$\|v[\mu] - v[\nu]\|_{C^0} \leq K W_p(\mu, \nu).$$

Generalized Wasserstein

$$\begin{cases} \partial_t \mu + \nabla \cdot (v[\mu]\, \mu) = h[\mu], \\ \mu_{|t=0} = \mu_0. \end{cases}$$

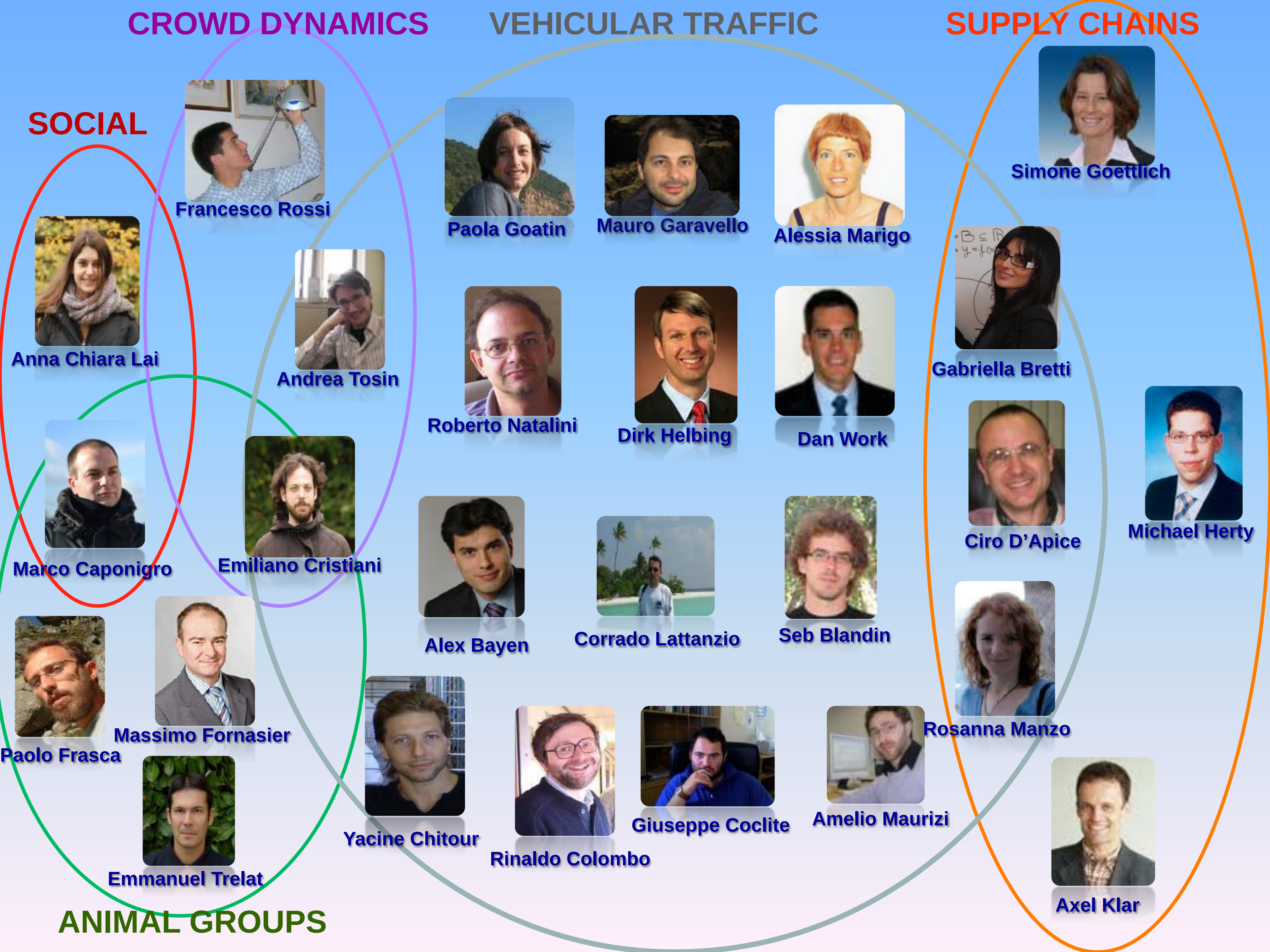
$$W_p^{a,b}(\mu,\nu) = \inf_{\substack{\tilde{\mu},\tilde{\nu} \in \mathcal{M}^p \\ |\tilde{\mu}| = |\tilde{\nu}|}} \left(a|\mu - \tilde{\mu}| + a|\nu - \tilde{\nu}| + bW_p(\tilde{\mu},\tilde{\nu})\right)$$

$$\mathcal{B}^{a,b}[\mu, v, h] := a^2 \left(\int_0^1 dt \left(\int_{\mathbb{R}^d} d|h_t| \right) \right)^2 + b^2 \int_0^1 dt \left(\int_{\mathbb{R}^d} d\mu_t\, |v_t|^2 \right).$$

$$T_2^{a,b}(\mu,\nu) = \inf_{\tilde{\mu},\tilde{\nu} \in \mathcal{M},\, |\tilde{\mu}| = |\tilde{\nu}|} a^2 \left(|\mu - \tilde{\mu}| + |\nu - \tilde{\nu}|\right)^2 + b^2 W_2^2(\tilde{\mu},\tilde{\nu})$$

$$T_2^{a,b}(\mu_0,\mu_1) = \inf \{ \mathcal{B}^{a,b}[\mu, v, h] \mid \mu \text{ is a solution of (1)}$$

$$\text{with vector field } v, \text{ source } h \text{ and } \mu_{|t=0} = \mu_0, \mu_{|t=1} = \mu_1 \}$$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